The Moon

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The Moon

Introduction:

The moon is at a distance of about 2,40,000 miles (384403 km) from the earth and as such it is the nearest celestial neighbor to the earth. The linear diameter of the moon is about 2163 miles (3476 km) and its mass is about 1/81 times that of the earth, the angular diameter of the moon is about 30 ^{l} and its mean horizontal parallax is about 57 ^{l}.

The moon is a satellite of the earth. It moves around the earth following the laws of Kepler. The lunar orbit is at an angle of about 5^{o} 8^{l} to the ecliptic and its eccentricity is 1/18. The two points of intersection of the lunar orbit and ecliptic are called the nodes of lunar orbit.

The point where the moon crosses the ecliptic in going north is called the ascending node and the other point where the moon crosses the ecliptic in going south is called the descending node. The line joining the nodes of the lunar orbit is called the nodal line and it is the line of intersection of the planes of the orbit of the moon and the ecliptic.

The moon is not a self-luminous body. It shines in the light it receives from the sun.

Sidereal month:

The sidereal month is the periods of one complete revolution of the moon around the earth relative to any fixed star. It is about 27 1/3 days (27 days , 7 hours, 43 minutes).

Synodic month:

The periods of one complete revolution of the moon around the earth relative to the sun is called a synodic month. It is also called a lunar month or a lunation. It is about 29 ½ days (29 days, 12 hours, 44 minutes).

To find the relation between sidereal and synodic months:

The Moon's sidereal orbital period (the sidereal month) is ~27.3 days; this is the time interval that the Moon takes to orbit 360° around the Earth relative to the "fixed" stars.

The period of the lunar phases (the synodic month), e.g. the full moon to full moon period, is longer at ~29.5 days.

At New Moon, the Moon is aligned with the Sun. During the lunar month the Earth orbits (revolves) ~30° around the Sun and the Moon orbits (revolves) ~390° to align with the Sun again.

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If $\mathbf{P}_{\mathbf{SID}}$ is the Moon's sidereal orbital period (in days),then the Moon revolves around the Earth at a rate of $360 / \mathbf{P}_{\mathbf{SID}}$ ° per day. So in a time interval **t** days, the Moon revolves around the Earth $\mathbf{t} \times 360 / \mathbf{P}_{\mathbf{SID}}$ °.

If $\mathbf{P}_{\mathbf{YEAR}}$ is the Earth's sidereal orbital period (in days), then the Earth revolves around the Sun at a rate of $360 / \mathbf{P}_{\mathbf{YEAR}}$ ° per day and so in **t** days the Earth revolves around the Sun $\mathbf{t} \times 360 / \mathbf{P}_{\mathbf{YEAR}}$ °.

We wish to determine the time interval when the angle that the Moon has revolved around the Earth **minus 360°** is equal to the angle the Earth has revolved around the Sun.

Hence $\mathbf{t} \times 360 / \mathbf{P}_{\mathbf{SID}} - 360 = \mathbf{t} \times 360 / \mathbf{P}_{\mathbf{YEAR}}$

Divide by $\mathbf{t} \times 360$, so

$1 / \mathbf{P}_{\mathbf{SID}} - 1 / \mathbf{t} = 1 / \mathbf{P}_{\mathbf{YEAR}}$

By definition $\mathbf{t} = \mathbf{P}_{\mathbf{SYN}}$.

Now $\mathbf{P}_{\mathbf{SID}}$ = 27.32158days, $\mathbf{P}_{\mathbf{YEAR}}$ = 365.25636days,

hence $\mathbf{P}_{\mathbf{SYN}}$ = 29.53049 days.

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Elongation:

The **elongation** of moon at any instant is the difference between the longitudes of the sun and moon. It is said to be east or west according as the moon is on the eastern or western side of the sun.

Let E be the earth and S the position of the sun on the ecliptic . Let M be the position of moon in its orbit and M1 the foot of the perpendicular from it to the ecliptic.

Elongation of moon = Longitude of moon – longitude of sun

$$= \chi M_1 - \chi S = SM_1 = \angle SEM_1$$

If the inclination of the lunar orbit be neglected and the moon were taken in the ecliptic then the elongation of the moon is the angles subtended by sun and moon at the earth.

Conjunction:

Moon is said to be in **Conjunction** with the sun if it seen from the earth in the same direction as the sun. At conjunction the elongation of moon is zero. Conjunction takes place on new moon days.

Opposition:

Moon is said to be in **opposition** with the sun when it is seen from the earth in the direction opposite to that of the sun. At opposition the elongation of the moon is 180^{o}. Opposition takes place on full moon days.

Quadratures:

The moon is said to be in a **Quadrature** if its elongation is 90^{o} . The quadrature is said to be east or west according as the moon is on the eastern or western side of the sun. The eastern and western quadrature position are also known as the first and third quadraters respectively.

Daily motion of the moon:

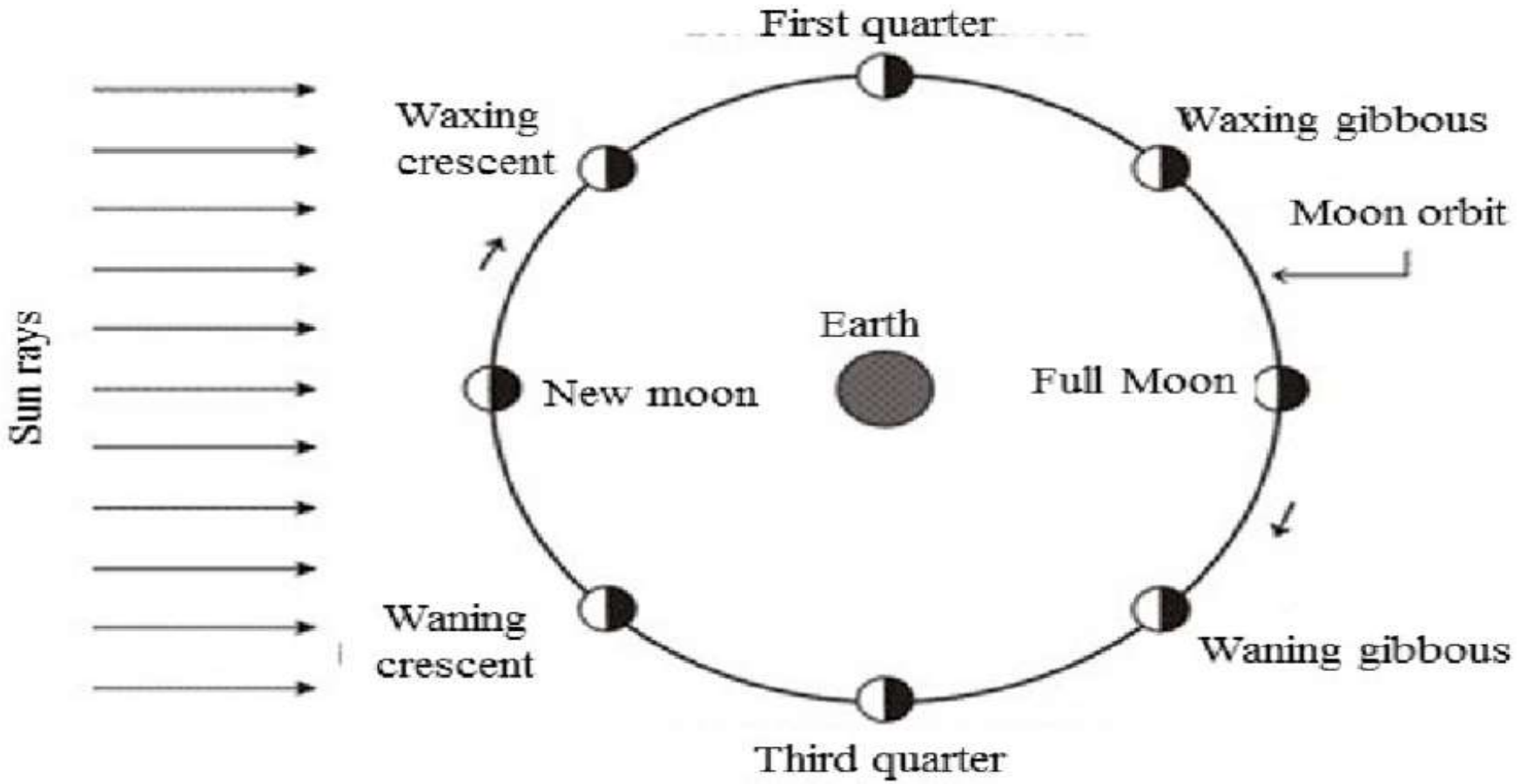
In a lunation of 29 ½ days, the moon separates through 360^{o} from the sun eastern relative to the earth. Therefore the average angular diameter of moon is also 30^{l}. Therefore the moon moves through an arc equal to its angular diameter every hour.

Age of moon:

The age of moon on any day is the number of days that has elapsed since the previous new moon day. It is measured at midnight. The age of moon multiplied by 12.2^{o} gives its distance east of sun, that is the eastern elongation of the moon.

Successive phases of moon:

As the moon moves along its orbit around the earth its hemisphere turned towards the sun is illuminated. For an observer on the earth only that part of the hemisphere turned towards the earth is visible. Therefore the whole illuminated portion of the moon is not visible and it is seen in different shapes. These are called the successive phases of moon.


First quarter
Waxing crescent
Waxing gibbous
Moon orbit
Sun rays
Earth
New moon
Full Moon
Waning crescent
Waning gibbous
Third quarter

When the moon is at M_1 in conjunction with the sun, only the non illuminated portion of the moon is turned towards the earth. Therefore the moon is not visible and it is said to be new, M_2 is a position of the moon within seven days of age. At this position only a small portion of the illuminated surface of moon is visible. It is then said to be crescent and is visible in the west after sun set. M_3 is the position of the moon when it is at 90^o from the sun. At this position it is seen as a bright semi circle. The moon is said to be dichotomized and it is at the first quarter or quadrature east. M_4 is a position of moon after the first quarter. At this position more than half the illuminated surface of moon is visible. The moon is said to be gibbous. M_5 is the position of moon when it is in opposition with the sun. At this position the moon is seen as a bright circular disc and the moon is said to be full.

Thus in half a lunation the moon exhibits all phases from new to full. In the remaining half lunation the phases are repeated in the reverse order from full to new.

When the moon is in conjunction or opposition with the sun, that is when it is new or full it is said to be in syzygy. From new moon to full moon the moon is said to be waxing and from full moon to new moon it is said to be waning.


thank you